Handout 6

1 The Household Problem

There is a continuum of households of measure one in each country. Home households maximize utility subject to their budget constraint, and the law of motion for capital

$$\max_{[C_t(h),W_t(h),Mb_t(h),I_t(h),K_{t+1}(h),B_{Dt+1}(h),B_{Ft+1}(h)]]} E_t \sum_{j=0}^{\infty} \beta^j \left(U(C_{t+j}(h), C_{t+j-1}(h))\right.$$
$$\left. V(L_{t+j}(h)) + \nu(\frac{Mb_t(h)}{P_{c,t}})\right) + \beta^j \lambda_{t+j}(h) \left[\Pi_t(h) + T_{t+j}(h) + W_{t+j}(h)L_{t+j}(h)\right.$$
$$+R_{k,t+j}K_{t+j}(h) - \frac{1}{2}\psi_k P_{It+j}K_{t+j}(h)\left(\frac{I_{t+j}(h)}{K_{t+j}(h)} - \delta\right)^2 - \frac{1}{2}\psi_I P_{It+j}\frac{(I_{t+j}(h) - I_{t+j-1}(h))^2}{I_{t+j-1}(h)}$$
$$-P_{Ct+j}C_{t+j}(h) - P_{It+j}I_{t+j}(h) - e_t Pb_t^* B_{F,t+1}(h) + e_t B_{F,t}(h)$$
$$\left. - \int_s \psi_{t+j+1,t+j}B_{Dt+j+1}(h) + B_{Dt+j}(h) - Mb_{t+j}(h) + Mb_{t+j-1}(h)\right]$$
$$+\beta^j Q_{t+j}(h)\left[(1-\delta)K_{t+j}(h) + I_{t+j}(h) - K_{t+j+1}(h)\right],$$

and subject to the labor demand schedule $L_t(h) = L_t\left(\frac{W_t(h)}{W_t}\right)^{-\frac{1+\theta_w}{\theta_w}}$.

We now have a few more terms, relative to the closed economy setup. The main difference is that we have a proliferation of prices, P_{Ct}, P_{It}, reflecting that investment and consumption are composites of the domestic and the foreign final goods. The asset structure is such that we have complete financial markets domestically and incomplete markets across countries. We also have P_{Bt}^* the price, in foreign currency, of the foreign bond. Finally, the term e_t is the nominal exchange rate, translating foreign currency units into domestic currency units.

1.1 First-Order Condition for Intermediate Prices of Export Goods

The pricing decisions of exporting firms are subject to Calvo-style contracts. In any period, an intermediate firm f can renew its price $P_{Mt}^*(f)$ set in the foreign currency with probability $1-\xi_m$. If a firm obtains the Calvo signal in period t, but not in any of the periods between t and $t+j$, then the firm resets its price according to $P_{Mt+s}(f) = P_{Mt}\pi^{*s}$, for all s between 1 and j.

Below, we consider the pricing decision of a firm that gets to renew its price in period t. Its profit maximization problem can then be written as:

$$\max_{[P^*_{Mt}(f)]} E_t \sum_{j=0}^{\infty} \xi_m^j \psi_{t+j,t} \left[(1+\tau_p) e_{t+j} P^*_{Mt}(f) {\pi^*}^j M^*_{t+j}(f) - \Sigma_{t+j} M^*_{t+j}(f) \right]. \tag{1}$$

Manipulations analogous to those for the first-order condition for the price of domestic intermediate goods in Handout 5 lead to:

$$\hat{\pi}^*_{Mt} = \beta \hat{\pi}^*_{Mt+1} + \kappa_p \left(\hat{\sigma}_t - \hat{e}_t - \hat{P^*_{Mt}} + \hat{P}_t \right) \tag{2}$$

where $\kappa_p = \frac{(1-\beta\xi_m)(1-\xi_m)}{\xi_m}$. Notice that $\hat{e}_t + \hat{P^*_{Mt}} - \hat{P}_t$ is the relative price of home exports (equivalently, foreign imports).