

## Homework 3

**Question 1**

Consider the model with imperfect financial markets described in handout 3 and the dynare model file associated with it (*completemarkets.mod*). Modify it to allow for country-specific temporary shocks to the growth of labor-augmenting technology following the instructions below.

In order to allow for growth rate shocks of labor-augmenting technology, we need to perform some transformations of the variables we use to describe the model, just as we would do for a closed-economy stochastic growth model. If the growth rate shocks affect both countries equally, these transformations will ensure that the transformed variables eventually return to the initial steady state. These transformations will also reduce the deviations from the initial steady states in response to country-specific temporary growth rate shocks.

Let  $z_t = e^{n_t}$ . Define the following variables:

$$\begin{aligned} \tilde{C}_t &= \frac{C_t}{z_t} & \tilde{\lambda}_t &= \lambda_t z_t & \tilde{\gamma}_t &= \gamma_t z_t & \tilde{Y}_t &= \frac{Y_t}{z_t} \\ \tilde{I}_t &= \frac{I_t}{z_t} & \tilde{K}_t &= \frac{K_t}{z_t} & \tilde{w}_t &= \frac{w_t}{z_t} & \tilde{C}_{Dt} &= \frac{C_{Dt}}{z_t} \\ \tilde{C}_{Mt} &= \frac{C_{Mt}}{z_t} & \tilde{B}_{Ft} &= \frac{B_{Ft}}{z_t} \end{aligned}$$

Let the equivalent foreign variables be rescaled by  $z_t^* \equiv e^{N^*t}$ . Create a new model file expressing the necessary conditions for an equilibrium in terms of the transformed variables above and the remaining non-transformed variables. Use the above transformations to get rid of any  $z_t$  and  $z_t^*$  terms. Replace the stochastic processes for  $N_t$  and  $N_t^*$  with AR(1) process for  $\mu_{nt}$  and  $\mu_{nt}^*$ , defined respectively as  $\frac{e^{N_t}}{e^{N_{t-1}}}$  and  $\frac{e^{N_t^*}}{e^{N_{t-1}^*}}$ .

Notice that in the inter-temporal equations you will have to introduce the growth rate terms  $\mu_{nt}$  or  $\mu_{nt}^*$ , as appropriate. Furthermore, in some of the equations, you will need to introduce a term for relative size of technology in the home and foreign country.

Assume that  $\mu_{nt}$  and  $\mu_{nt}^*$  are both zero in steady state. Furthermore, assume that  $N_t$  and  $N_{t}^*$  are 0 in the initial steady state.

Reproduce the responses reported by the program *call\_incomplete* for the case of a 1% shock to  $\mu_{nt}$  when its AR(1) coefficient is 0 and when its AR(1) coefficient is 0.9.

Notice that to plot the responses of the un-transformed variables you will have to manipulate the output of the program *makeirf*. You'll have to undo the transformation before producing the plots.

### **Question 2**

How does the value that you choose for the trade elasticity affect the response of the trade balance to the growth rate shock in question 1?

### **Question 3**

Follow the instructions below to determine how the relative size of the home and foreign country affect the response of the home and foreign trade balance to the growth rate shock in question 1. Also, report how country size affects the response of the home and foreign risk-free rate. Does this change when the elasticity of substitution between home and foreign goods is very high?

In order to allow for countries of different sizes, let the continuum of households have different measures in the home and the foreign country. You can then normalize the state variables for each country by its population size to yield per capita quantities. As a result, you will have to add a parameter for relative population sizes in the equations where both home and foreign variables enter at the same time (e.g. the net-foreign-asset equation).

### **Question 4**

Introduce habit persistence in consumption by modifying the utility function as follows:

$\sum_{j=0}^{\infty} \beta^j \left( (1 - \phi_c) \log(C_t - \phi_c C_{t-1}^A) + V(L_{t+j}) \right)$ . The lagged value of consumption is for the aggregate economy, rather than for an individual household (habits are external).

How does varying the degree of habit persistence affect the response of the trade balance to the growth shocks in question 1?